

LOGIC EXERCISES: WEEK 4

QUESTION 1

1. Using the same dictionary as in week 3, question 3, formalise the following English claims in L_2 , and render the L_2 claims into idiomatic English.

- i. No drinker likes all the smokers.
- ii. No drinker likes any of the smokers.
- iii. If everyone smokes, Beth does.
- iv. If anyone smokes, Beth does.
- v. If all the smokers like Cecil, then none of the drinkers do.
- vi. If nobody smokes, then nobody smokes and drinks.
- vii. Only Beth and Di like Albert.
- viii. Albert likes himself, but he doesn't like anybody else.
- ix. All the smokers like themselves, but none of the drinkers do.
- x. Some of the smokers like some of the drinkers, but no smoker likes them all.
- xi. $\forall x \exists y Lxy$
- xii. $\exists y \forall x Lxy$
- xiii. $\forall x (Lxa \rightarrow Lba)$
- xiv. $(\forall x Lxa \rightarrow Lba)$
- xv. $(\forall x \neg Lax \rightarrow \neg \exists x (Lax \wedge Sx))$

QUESTION 2

How many elements are there in the following sets?

- i. $\{1, 2, 3, 2, 1\}$
- ii. $\{1, 2, 3, \text{the first prime number}\}$
- iii. $\{1, 2, 3, \{1\}\}$
- iv. $\{1, 2, \langle 1, 2 \rangle\}$
- v. $\{\langle 1, 2 \rangle, \langle 1, 2 \rangle\}$
- vi. $\{\langle 1, 2 \rangle, \langle 2, 1 \rangle\}$
- vii. \emptyset
- viii. $\{\emptyset\}$
- ix. $\{x : x \text{ is a subject offered with philosophy at Oxford}\}$
- x. $\{\langle x, y \rangle : x \text{ is a pope and } y \text{ is a current US president}\}$

QUESTION 3

Determine whether the following binary relations (R) are reflexive, symmetric, asymmetric, and/or transitive on the given sets (S), and whether or not they are functions:

- i. $R = \{\langle x, y \rangle : y \text{ is } x\text{'s biological father}\}$, $S = \{x : x \text{ is a living human being}\}$
- ii. $R = \{\langle x, y \rangle : x \text{ is taller than } y\}$, $S = \{x : x \text{ is a living human being}\}$

- iii. $R = \{ \langle x, y \rangle : x \text{ was born in the same decade as } y \}, S = \text{ditto}$
- iv. $R = \{ \langle x, y \rangle : x \text{ was born within ten years of } y \}, S = \text{ditto}$
- v. $R = S = \emptyset$

QUESTION 4

Provide binary relations R and sets S such that R has the following properties on S (don't reuse any of the examples above):

- i. transitive
- ii. not symmetric, not transitive
- iii. reflexive, symmetric, transitive
- iv. reflexive, symmetric, not transitive
- v. irreflexive, asymmetric, transitive
- vi. not reflexive, not symmetric, transitive
- vii. not reflexive, not symmetric, not transitive
- viii. reflexive, not symmetric, not transitive
- ix. reflexive, antisymmetric, transitive
- x. symmetric, asymmetric